Appendix A Solve the utility function

The expected utility function can be written as

$$U = w(p)\cdot v(x) + w(q)\cdot v(y) = w(p)\cdot f(Q)^{\sigma} - w(q)\cdot \lambda[-C(Q)]^{\sigma} \qquad \text{(I-1)}$$

Since $w(q) = w(1) = 1$, we have $U = w(p)\cdot f(Q)^{\sigma} - \lambda[-C(Q)]^{\sigma}$, where $w(p) = \exp[-(-\ln p)^{\alpha}]$. To solve equation (I-1), or to get farmer's optimal fertilizer use level, we should have $\frac{dU}{dQ} = 0$. From equation (I-1), we have

$$\frac{dU}{dQ} = w(p)\cdot \sigma f(Q)^{\sigma-1}\cdot f'(Q) - \lambda\sigma[-C(Q)]^{\sigma-1}\cdot C'(Q)$$

Plug $x = f(Q) = \mathrm{A}*\mathrm{Q}^{B}$ and $y = C(Q) = -\mathrm{c}*\mathrm{Q}$ into the above equation to get

$$\frac{dU}{dQ} = w(p)\cdot \sigma[AQ^{B}]^{\sigma-1}\cdot ABQ^{B-1} + \lambda\sigma(cQ)^{\sigma-1}\cdot(-c) \qquad \text{(I-2)}$$

If we set $\frac{dU}{dQ} = 0$, we get $w(p)\cdot \sigma[AQ^{B}]^{\sigma-1}\cdot ABQ^{B-1} = c\lambda\sigma(cQ)^{\sigma-1}$. Solving this equation, we get

$$Q^{*} = \left(\frac{\lambda\cdot c^{\sigma}}{B\cdot w(\mathrm{p})\cdot A^{\sigma}}\right)^{\frac{1}{(B-1)\sigma}} = \left(\frac{\lambda}{B\cdot w(\mathrm{p})}\right)^{\frac{1}{(B-1)\sigma}}\cdot\left(\frac{c}{A}\right)^{\frac{1}{B-1}} \qquad \text{(I-3)}$$

Since $w(p) = \exp[-(-\ln p)^{\alpha}]$, the equation (I-3) can be re-written as

$$Q^{*} = \left(\frac{c}{A}\right)^{\frac{1}{B-1}}\cdot\left(\frac{\lambda}{B}\right)^{\frac{1}{(B-1)\sigma}}\cdot \mathrm{e}^{\frac{(-\ln \mathrm{p})^{\alpha}}{(B-1)\sigma}} \qquad \text{(I-4)}$$

Appendix B The impact of risk preference on fertilizer use

As shown in equation (I-3), the farmer's optimal level of fertilizer use is

$$Q^* = (\frac{\lambda}{B \cdot w(\mathrm{p})})^{\frac{1}{(B-1)\sigma}} \cdot (\frac{c}{A})^{\frac{1}{B-1}}$$

In order to have a good understanding of this equation, we define $H(p) = \frac{\lambda}{w(p)B}$.

Since $\lambda > 0$, $B > 0$, and $w(p) = \exp[-(-\ln p)^{\alpha}] > 0$, we have H(p)>0.

We then define $(\frac{c}{A})^{\frac{1}{B-1}} = k$. Since c>0, B>0, and A>0, we have $k > 0$. Then optimal fertilizer use level equation can be re-written as

$$Q^* = k \cdot H(p)^{\frac{1}{(B-1)\sigma}} \quad \text{(II-1)}$$

In order to show the impact of farmer's risk preference on the optimal level of fertilizer use, we get

$$\frac{dQ^*}{d\sigma} = k \cdot H(p)^{\frac{1}{(B-1)\sigma}} \cdot \ln H(p) \cdot \frac{1}{B-1} \frac{-1}{\sigma^2} = \frac{k \cdot \ln H(p)}{(1-B)\sigma^2} \cdot H(p)^{\frac{1}{(B-1)\sigma}} \quad \text{(II-2)}$$

Since $\frac{k}{(1-B)\sigma^2} > 0$, $H(p)^{\frac{1}{(B-1)\sigma}} > 0$, the sign of $\frac{dQ}{d\sigma}$ is the same as the sign of lnH(p) (where $H(p) = \frac{\lambda}{w(p)B} = \frac{\lambda}{B} \cdot \mathrm{e}^{(-\ln \mathrm{p})^{\alpha}}$). If lnH(p)>0, then $\frac{dQ}{d\sigma}$>0, and vice versa.

In the following, we are going to discuss the sign of lnH(p) for two cases.

Case I. $B \leq \lambda$

If $B \leq \lambda$, we have $\frac{\lambda}{B} \geq 1$. And because $\mathrm{e}^{(-\ln p)^{\alpha}} > 1$, we have $H(p) > 1$. So we have $\ln H(p)$ >0. And we have $\frac{dQ^*}{d\sigma} > 0$.

Case II. $B > \lambda$

Before discussing the sign of $\ln H(p)$, we need to discuss the magnitude of p. If $p < \mathrm{e}^{-(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}}$, we have $\ln p < -(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}$. Then we have $-\ln p > (\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}$. Then we have $(-\ln p)^{\alpha} > \ln\frac{B}{\lambda}$. Finally, we have $\exp[(-\ln p)^{\alpha}] > \frac{B}{\lambda}$. As a result, we have $H(p) = \frac{\lambda}{w(p)B} = \frac{\lambda}{B}\cdot\exp[(-\ln p)^{\alpha}] > 1$. Since $\ln H(p) > 0$, according to equation (II-2), we have $\frac{dQ^*}{d\sigma} > 0$.

If $p \geq \mathrm{e}^{-(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}}$, we have $\ln p \geq -(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}$, and $-\ln p \leq (\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}$. So we have $(-\ln p)^{\alpha} \leq \ln\frac{B}{\lambda}$, and $\exp[(-\ln p)^{\alpha}] \leq \frac{B}{\lambda}$. As a result, we have $H(p) = \frac{\lambda}{w(p)B} = \frac{\lambda}{B}\cdot\exp[(-\ln p)^{\alpha}] \leq 1$, and so $\ln H(p) < 0$. According to equation (II-2), we have $\frac{dQ^*}{d\sigma} < 0$.

To summarize, we have

$$\frac{dQ^*}{d\sigma} > 0\text{, if } \begin{cases} B \leq \lambda \\ B > \lambda \textit{ and } p < e^{-(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}} \end{cases}$$

$$\frac{dQ^*}{d\sigma} \leq 0\text{, if } B > \lambda \text{ and } p \geq \mathrm{e}^{-(\ln\frac{B}{\lambda})^{\frac{1}{\alpha}}} \qquad \text{(II-3)}$$